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Curtin University
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EDP 343: Assessment 3
Child Study Report

Due: 17 July 2016

Prepared for:
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Curtin University
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1.0 Introduction

This report describes and explains the process undertaken for advancing a child’s conceptual understanding of measurement in mathematics. Its objective is to accomplish this advancement for a Year 4 child, hereinafter referred to as Grace, through three interlocking stages. Firstly, Grace’s level of understanding of the key principles of measurement was diagnosed. Secondly, the findings of the diagnosis provided the basis for the development of learning opportunities. Thirdly, the tutoring sessions, using constructivist principles, focused on elevating Grace’s grasp of the measurement concepts in most need of attention, namely visualisation, spatial reasoning, area and perimeter. The report is structured in this order, and concludes with recommendations regarding the next areas of measurement that would best extend Grace’s conceptual understanding, and a summary of Grace’s development.

1.1 Background

This report was conducted after discussions with Grace’s parents and the completion of a consent form (Appendix A). In class, Grace was learning about fractions, area, perimeter and volume. The assessment included questions from the Years 3 and 5 sample NAPLAN numeracy tests (National Assessment Program, 2013). These questions covered measurement concepts that Grace would have already learned and concepts she was yet to learn. Further, two Diagnostic Tasks were selected from the First Steps in Mathematics program (Department of Education WA [DEWA], 2013a, pp. 140-156). These were Tiling Problem and Block Towers. A third task was devised to diagnose Grace’s understanding of perimeter and area.

The two Content Descriptors of the Year 4 Mathematics curriculum covered by this diagnostic assessment are:

- ACMMG290 – “Compare objects using familiar metric units of area and volume” (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2016);
- ACMMG087 – “Compare the areas of regular and irregular shapes by informal means” (ACARA, 2016).
2.0 Diagnostic results

Grace could not identify the correct number of cubes within a solid stack [Figure 1], indicating a difficulty with “visualisation and spatial reasoning” (Reys et al., 2012, p. 393). She counted only the cubes she could see. This was a concern because this was a Year 3 question, and “spatial sense” can impact a student’s ability to understand fractions (Booker, Bond, Sparrow & Swan, 2014, p. 52).

Grace confused area with perimeter (Reys et al., 2012, p. 423) [Figure 2]. While Grace answered a separate perimeter question correctly [Figure 3] it was only attained after considerable difficulty.
In the *Tiling Problem*, Grace correctly calculated the area [Figure 4]. The measurement was not made multiplicatively, however, and therefore Grace did not recognise the “spatial structuring” of the shape (DEWA, 2013b). In *Block Towers* [Figure 5], Grace recognised each block as “slices” (DEWA, 2013c, p. 14) however she used an additive approach (Booker et al., 2014, p. 196). Grace described perimeter as an “outline” but drew a perimeter as an open shape, thereby demonstrating a lack of conceptual awareness [Figure 6].
3.0 Lesson planning

The diagnostic assessment determined the focus of the six lessons:

- Spatial reasoning as it relates to measurement
- Perimeter and its relationship to area

<table>
<thead>
<tr>
<th>What Grace knows</th>
<th>What Grace needs to know</th>
</tr>
</thead>
</table>
| Measures what she can see | - Identify “relationships between … lengths of edges and … volume” (DEWA, 2013c, p. 11)  
- Working towards Level 4 of Indirect Measure (DEWA, 2013c, p. 10) |
| Uses additive approach | - “Multiplicative thinking” (Booker et al., 2014, p. 266)  
- Working towards Level 4 of Indirect Measure (DEWA, 2013c, p. 13)  
- “understand and use the multiplicative features of the array structure” (DEWA, 2013b, p. 166) |
| Confuses area and perimeter | - “Distinguish perimeter from area” (DEWA, 2013b, p. 10)  
- Working towards Level 4 of Understanding Units (DEWA, 2013b, p. 10)  
- Understand and apply correct calculations for the task (DEWA, 2013c, p. 54)  
- Working towards Level 3 of Indirect Measure (DEWA, 2013c, p. 10) |

Each lesson was created to suit Grace’s mathematical ability (Reys et al., 2012, p. 24). The diagnostic helped identify her “zone of proximal development” (Vygotsky, 1978, p. 87) and shaped the emphasis of the lessons. The first lesson was a response to the diagnostic tasks, and each successive lesson was crafted based upon Grace’s progress.

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Topic</th>
<th>Key Understanding</th>
<th>Activity</th>
<th>Consolidation</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>Spatial sense</td>
<td>Indirect Measure: KU 1, 4</td>
<td>Labelling prism faces</td>
<td>Build a prism to match a drawing</td>
</tr>
<tr>
<td>Two</td>
<td>Spatial sense</td>
<td>Indirect Measure: KU 1, 4</td>
<td>Calculate volume and build the diagram</td>
<td>Draw a built model</td>
</tr>
</tbody>
</table>
Three

<table>
<thead>
<tr>
<th>Spatial sense</th>
<th>Indirect measure:</th>
<th>Measuring and</th>
<th>Build a model from</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KU 1, 4</td>
<td>labelling cube stack diagrams</td>
<td>instructions</td>
</tr>
</tbody>
</table>

Four

<table>
<thead>
<tr>
<th>Area and perimeter</th>
<th>Indirect Measure:</th>
<th>Nine Tiles</th>
<th>Online area and perimeter challenges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KU 1</td>
<td>Units: KU 1, 3</td>
<td>Which paddock is bigger?</td>
</tr>
</tbody>
</table>

Five

<table>
<thead>
<tr>
<th>Area and perimeter</th>
<th>Indirect Measure:</th>
<th>Fertiliser problem</th>
<th>Painting the wall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KU 1, 4</td>
<td></td>
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</tbody>
</table>

Six

<table>
<thead>
<tr>
<th>Area and perimeter</th>
<th>Indirect Measure:</th>
<th>Farm fences</th>
<th>Farm problem</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>KU 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Units: KU 1, 3</td>
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</tbody>
</table>

The pedagogy for these lessons required careful attention. The purpose was to strengthen and extend Grace’s conceptual understanding of measurement in the key areas using a constructivist approach. To this end, tasks were designed that gave Grace control over her learning, allowed her to build upon and extend her current understandings, and engaged her through “discussion … and negotiation of meanings” (Booker et al., 2014, p. 15). Importantly, each lesson was designed to be meaningful to ensure motivation for learning (Annenburg Media, 2016), and emphasise process over product (Sullivan & Millsom, 2009, p. 17) to monitor Grace’s grasp of the concepts behind each problem.

3.1 Resources

The resources for the three lessons about spatial reasoning were selected for their ability to extend Grace’s understanding. The use of drawings to be built, and built objects to be drawn, for example, boosted Grace’s ability to manipulate images mentally (Reys et al., 2012, p. 395). Manipulatives such as Lego blocks were also chosen for their impact on helping Grace make sense of mathematics concepts (Booker et al., 2014, p. 21). Combining such resources over several lessons gave Grace opportunities to enhance her understanding (Lowe, 2013, p. 8) through repeated exposure.

Resources for lessons four to six were chosen to provide greater engagement and motivation for learning. For example, “real-world” tasks (Peter-Koop, 2005, p. 4), such as the Fertiliser and Painting the wall problems provided a purpose for the learning and a chance for Grace to apply her knowledge in different settings (Lowe, 2103, p. 8). Manipulatives, such as the Nine Tiles task, supported and underpinned the constructivist approach and served to deepen Grace’s area and perimeter knowledge. Finally, all lessons involved
“nonroutine problems” (Reys et al., 2012, p. 114) to avoid the mere application of rote-learned facts.

4.0 Tutoring sessions

4.1 Visualisation/spatial reasoning (Reys et al., 2012, p. 393)

Grace had to select a net that would match a numbered object built from cardboard [Figure 7]. Grace used correct mathematical language to identify the object as a “rectangular prism”, selected the correct net and explained her decision: “Because I thought about it, folded it in my head to match”.

Next, Grace labelled the net correctly, and folded it to form a rectangular prism to match the completed model [Figure 8]. She folded the net first so that it resembled the model, and then rotated both objects to determine the correct labels for the faces. Grace was asked to explain her thinking: “how to make this look like that and whether the numbers should go up this way or that way”. Significantly, the numbers and their orientation on each face matched the model [Figure 9].

Grace was next shown a rectangular prism diagram with labelled faces. She correctly labelled the hidden faces when viewing a rotated diagram of the prism [Figure 10] and explained she rotated the diagram in her head. Further, Grace was able to label 2D versions of the faces [Figure 11]. For consolidation, Grace labelled a net of the prism diagram by folding the net to match
the model, labelling each face [Figure 12], and assembling the model.

Grace was provided with the opportunity to extend her spatial reasoning abilities by counting cubes in a drawn stack [Figure 13], and then building it from Lego blocks. While she
incorrectly counted nine cubes ("because I can see nine cubes"), she recognised her error when building the diagram: "I'm pretty sure there are more blocks under there". She correctly measured 11 cubes in the stack and explained her answer: "because I have actually built the whole object" [Figure 14]. Grace displayed metacognitive ability when she reminded herself to "think what's underneath it or behind it". This was an important step in her conceptual understanding (Reys et al., 2012, p. 129).

When Grace viewed a stack of 21 Lego blocks she counted only 16 [Figure 15]. With some scaffolding she developed an efficient method for counting every block. She reminded herself to count the “unseen ones”, and made a record to ensure the accuracy of her answer (Annenburg Media, 2016) [Figure 16]. For consolidation, Grace drew a top view and a front view of the model. She then checked and labelled each diagram to confirm their accuracy [Figure 17].
When shown a new diagram of cube stacks [Figure 18], Grace correctly measured 13 blocks and explained “I counted the ones that you can’t see, actually think what’s underneath the ones you can see”. This demonstrated increasing spatial reasoning (Reys et al., 2012, p. 393) and metacognition. Further, Grace correctly labelled different views of 2D and 3D diagrams of a labelled stack [Figures 19-21]. Importantly she checked her answers for confirmation. Grace built the diagram using Lego blocks which confirmed and extended her “geometric thinking” (Reys et al., 2012, p. 395).
By way of consolidation she was given a task to build cube stacks with specific parameters (Reys et al., 2012, p. 119). Grace listed the parameters [Figure 22], constructed the stack [Figure 23], and drew the stack from a side elevation.

### 4.2 Area and perimeter

Development of these concepts commenced with the *Nine Tiles* activity (Booker et al., 2014, p. 425). Grace made some errors but persevered without assistance to maintain a constant perimeter while reducing the area [Figure 24]. Her recognition of the difference between one and two dimensional measurements provided a foundation for the remainder of the lesson. The *Which Paddock is Bigger?* task [Figure 25] (adapted from DEWA, 2013b, p. 32) began with Grace guessing that Paddock B was “bigger square metres, area”. When asked to confirm her guess she measured the perimeter of both shapes, thereby indicating continuing confusion over perimeter and area (Reys et al., 2012, p. 423). Some scaffolding was necessary to ensure an efficient and accurate measurement of both perimeters. To calculate the areas, Grace successfully divided each shape into quadrilaterals and calculated the area of each. She applied additive,
rather than multiplicative, thinking however, and some further scaffolding, with reference to arrays was necessary (Booker et al., 2014, p. 268) to extend her thinking. Her calculations confirmed her guess.

For consolidation purposes, Grace completed several perimeter and area activities online [Figures 26-27] (Birmingham Grid for Learning, 2016). She found these activities engaging and appeared to enjoy getting the answers right, which increased her motivation. She applied efficient processes, such as multiplying to calculate area, and required minimal scaffolding.

The Fertiliser Problem [Figure 28] required Grace to decide which problems to solve and how to solve them (Jones, 2003, p. 87). It required Grace to extend her “multiplicative thinking” (Siemon et al., 2011, p. 351) from the last lesson. Questions provided the necessary scaffolding, and Grace was able to identify and then order the problems as follows:

- How many ‘50gms’ are in 2kgs?
- How many square metres will 2kgs of fertiliser cover?
- How big is the front yard?
- Will there be enough fertiliser?

Grace applied “multiplicative thinking” (Siemon et al., 2011, p. 351) (division) to answer the first two questions. Grace selected a tape measure and explained she would need to first calculate the perimeter
of the yard to measure its area. Her measurements [Figure 29] were added to a perimeter diagram [Figure 30]. With the measurements recorded, Grace confronted the problem of calculating the area. She determined that if she multiplied the longest length by the longest width, she could subtract the “little bits” (smaller areas) from the measurement. She explained this would be more efficient than dividing the lawn into several areas and measuring each one. This was an important development in her thinking. She correctly calculated the area to be 68.75 m², and confirmed there would not be enough fertiliser [Figure 31].

In the next problem, Grace had to calculate the amount and cost of paint to cover a bedroom wall three times, requiring her to identify and solve several problems. Grace again applied a multiplicative approach, and with minimal scaffolding, employed successful processes to arrive at the answer [Figure 32]. Subsequently, Grace employed her efficient measuring strategy from the previous session to measure the perimeter accurately. She said this made her confident of measuring correctly. She accurately measured the perimeter as 72m, and then divided the farm into two rectangles, calculated the area of each using multiplication, and added the results to measure the area as 272m² [Figure 33].
To make the farm larger using the same perimeter, Grace was reminded of the *Nine Tiles* activity. She extended the farm from an L shape into a rectangle. Grace said that because the six metre and eight metre fence sections had been moved, not altered, the perimeter would remain 72m. To measure the area, Grace multiplied six by eight, and added 48 to 272 to confirm the new area as 320m$^2$.

Grace recognised the pattern (Sparrow & Swan, 2006, p. 138) and made a smaller L to create a smaller area using the same perimeter. She explained she could move the inside corner of the L to numerous places and the area would always be less than 320m$^2$ while maintaining the same perimeter [Figure 34]. Grace thereby demonstrated her conceptual understanding (Reys et al., 2012, p. 115). She created a farm with an area of 200m$^2$ and a perimeter of 72m. To make the smallest farm possible, Grace required minimal scaffolding to draw a farm measuring 1m metre by 35m [Figure 35].
The session concluded with a problem with specific limitations (Reys et al., 2012, p. 119) [Figure 36].

Sue has been told by the Council that the perimeter of her farm must be 64 metres, and the area of her farm must be 240m². Can you help Sue design her new farm?

Grace demonstrated her increased conceptual understanding throughout the task. Grace started by drawing a square farm and calculated each side to be 16m, but realised the area to be 256m². She said “I need to take off 16m²”. She removed a 4m x 4m section from the drawing to create a farm of the stipulated area and perimeter [Figure 37].

5.0 Progression

The next stage in Grace’s learning would include:

- Continued exposure to problems involving visualisation, spatial reasoning, and area;
- Recognising 3D objects from different angles and interpreting solids based on cross-sections (Reys et al., 2012, p. 395);
- Drawing and assembling nets of more complex polyhedrons (Booker et al., 2014, p. 477);
- Introducing the area of triangles (Booker et al., 2014, p. 422);
- Strengthening her Level 4 achievement in Indirect Measure, and working towards Level 5 (DEWA, 2013c, p. 10);

- Advancing to the “Quantifying Phase” of measurement (DEWA, 2013c, p. iii);

- Moving towards “location and transformation” of objects (Booker et al., 2014, p. 481) in accordance with the Australian Curriculum – ACMMG090 and ACMMG091 (ACARA, 2016); and

- Moving towards understanding angles in accordance with the Australian Curriculum – ACMMG089 (ACARA, 2016).

6.0 Conclusion

A child’s conceptual understanding of mathematical concepts can be strengthened and extended through the deployment of an effective three stage process of diagnosis, planning and tutoring. Importantly, the impact of the learning is enriched through the use of constructivist principles. In the sessions completed for this report, Grace was able to identify her own misconceptions, and was provided with opportunities, such as through manipulatives and meaningful activities, to create and adopt new understandings. Significantly, Grace was able to successfully apply her new learnings to future tasks. The first sessions extended Grace’s visualisation and spatial reasoning through engaging challenges that helped her make sense of the abstract and deepen her conceptual grasp. She applied increased spatial reasoning, developed efficient counting methods, and expressed awareness of her own thinking and learning. In the final sessions Grace developed efficient strategies for measuring area and perimeter, clearly understood their conceptual differences, and began thinking multiplicatively. Grace made considerable and demonstrable progress as a result of the tutoring. While continued exposure to these concepts is essential, she is now ready for new mathematical challenges in the field of measurement. Accordingly, the objective of this report has been met and serves as a basis for future teaching and learning.
References


Appendix A

Dear Parent,

The purpose of this letter is to introduce one of our teacher education students (pre-service teachers or PSTs) who would like to work with your child.

As part of their development as teachers, PSTs in Curtin’s Bachelor of Education program are enrolled in the unit Inquiry in the Mathematics Classroom EDPR3000. They are required to work with a child to learn about his/her understanding of a selected key mathematics concept, in this case aspects of measurement. This will involve a task-based interview or diagnostic assessment. On the basis of that assessment, our PSTs will design a teaching plan for six one-hour one-on-one tutoring sessions with your child. They will design or adapt activities that will enable them to assess the learning and level of understanding of the child on an ongoing basis. Activities and analysis of the children’s work will be based on the Australian Curriculum: Mathematics.

The PSTs will examine the data collected from their work and draw conclusions about the child’s understandings and skills. They will then submit a report about these assessments, along with a report of their learning as a teacher, as a formal assessment requirement of their mathematics education unit. In this process, and in the PSTs’ formal assignment submissions, the child will not be identified. That is, the child’s name, photos, or other features of the work that might identify him/her will not be used.

If you are happy for your child to participate in this small study and for his/her work to be used, please sign the form below and return it to the Curtin University Student. If you have questions about the study or activities please contact me by email (Susan.Brenchley@curtin.edu.au or paul.brown@curtin.edu.au).

Yours sincerely,

Susan Brenchley Unit Coordinator
Dr Paul Brown Unit Champion
May 2016

Name of Child: Audrey Ryan

I agree for my child to participate in the diagnostic assessment interview and series of tutoring sessions with a teacher education student, and I agree that work produced by my child may be used as part of a report about the activities.

Parent/Carer signature: [Signature]

Date: 4 June 2016